Four Methods for Quickly Establishing Orbital Systems

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THIS note describes the orbital transfer characteristics of four methods for establishing a system of equally spaced satellites in one circular orbit. These methods are compared on the bases of time to establish the system and propulsion requirements. It is assumed that all of the satellites, as a group, have been placed into a low-altitude circular parking orbit, which is coplanar with the target orbit. Thus, the problem consists of transferring a group of n satellites from a low-altitude circular orbit to a coplanar, higher-altitude circular orbit and spacing them at $360°/n$ intervals.

Phased Ellipse Method, Target Apogee (PE, TA)

This method may best be described by referring to Fig. 1. At $t = 0$, a velocity increment ΔV_1 sufficient to produce a transfer orbit, which at apogee will be tangent to the target orbit, is added to the n satellites (separately or collectively). At apogee, ΔV_2, which will result in the phased ellipse, is added to all but one $(n - 1)$ of the satellites. An increment $\Delta V_2 + \Delta V_3$, which will produce circular orbit velocity, is added to that one satellite. Thus, one satellite is in the target orbit, and $(n - 1)$ satellites are in the phased ellipse. At subsequent apogees of the phased ellipse, the $(n - 1)$ satellites are injected, with ΔV_3, one by one into the target orbit. After $(n - 1)$ revolutions in the phased ellipse, the system is established. Thus the total time required to establish the system is

$$T = (P_{TR}/2) + (n - 1)P_F \tag{1}$$

To assure the proper spacing in the target orbit, the period of the phased ellipse is selected as follows. Since the parking orbit is at low altitude, the perigee altitude of the phased ellipse probably will not be less than the parking orbit altitude; therefore

$$a_T \leq 2a_F - a_P \qquad \text{or } (a_F/a_T) \geq \tfrac{1}{2}[1 + (a_P/a_T)] \tag{2}$$

where a_T, a_F, and a_P are the semimajor axes of the target, phased and parking orbits, respectively. From the Kepler period equation

$$\frac{P_F}{P_T} \geq \left[\frac{1}{2}\left(1 + \frac{a_P}{a_T}\right)\right]^{3/2} = 0.354\left[1 + \frac{3}{2}\left(\frac{a_P}{a_T}\right) + \frac{3}{8}\left(\frac{a_P}{a_T}\right)^2 + \ldots\right] \tag{3}$$

Given values of a_P and a_T, lower bounds on a_F and P_F may be obtained. Note that the ratio of the phased ellipse period to the target period can never be less than 0.354. A further constraint on P_F/P_T is that

$$P_F/P_T = (n - i)/n \tag{4}$$

where i is any integer between 1 and $(n - 1)$ that yields values for $(n - i)$ and n, which are relative primes. For example, if $n = 4$, then i can be 1 or 3 but not 2, because 2 would produce a 0°, 180°, 0°, 180° pattern rather than the desired 0°, 90°, 180°, 270° pattern. Equation (4) implicitly assumes that a satellite will be injected into the target orbit at every apogee passage in the phased ellipse (not every second or third revolution), so that the time to establish the system will be reduced to a minimum. Equation (4) also constrains the phased ellipse to be internal to the target orbit (i.e., $a_F < a_T$) in order to economize on ΔV requirements as well as time. Given a_P, a_T, and n, one determines, from Eq. (4), the allowable values of P_F/P_T which are greater than the lower bound specified by Eq. (3); the smallest allowable P_F/P_T will yield the desired minimum time. Since only internal phased ellipses are being considered, the ΔV_{total} requirements for the various allowable values of P_F/P_T are identical.

As an example, consider a parking orbit altitude of 150 naut miles, a target orbit altitude of 8000 naut miles, and a system composed of four satellites. If this system were equatorial, the zonal band between ±65° latitude would be covered continuously.[1] Such a system might be very attractive for communication purposes. Since the radius of the earth R is approximately 3440 naut miles, then a_P = 3590 naut miles (P_P = 1.50 hr), a_T = 11,440 naut miles (P_T = 8.52 hr), and a_P/a_T = 0.314. From Eq. (4), allowable values of P_F/P_T are $\frac{1}{4}$ and $\frac{3}{4}$, but $P_F/P_T = \frac{1}{4}$ is eliminated because the lower bound on P_F/P_T from Eq. (3) is 0.532. Thus, using $P_F/P_T = \frac{3}{4}$,

ΔV_1 = 5930 fps, ΔV_2 = 2780 fps } can be supplied by boost vehicle

ΔV_3 = 1610 fps must be supplied by satellite

$$T = (4.54/2) + (3)(6.39) = 21.44 \text{ hr [from Eq. (1)]}$$

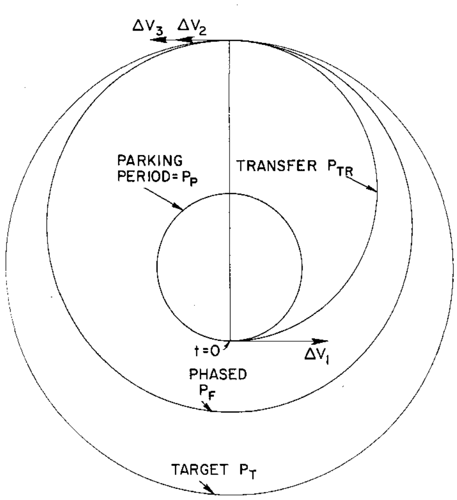

Fig. 1 Schematic of phased ellipse method (target apogee).

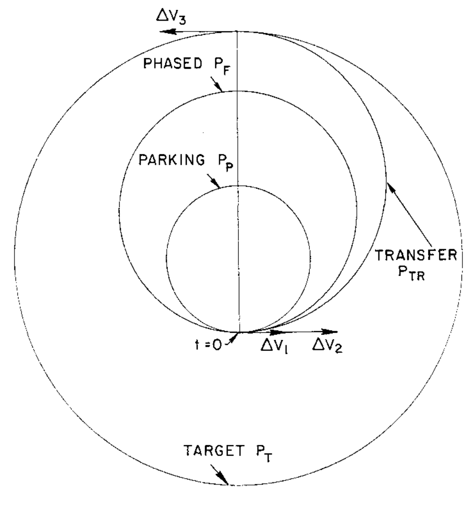

Fig. 2 Schematic of phased ellipse method (parking perigee).

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where

$$a_{TR} = (3590 + 11440)/2 = 7515 \text{ naut miles}$$

$$P_{TR} = P_P(a_{TR}/a_P)^{3/2} = (1.50)(7515/3590)^{3/2} = 4.54 \text{ hr}$$

and

$$P_F = \tfrac{3}{4}P_T = \tfrac{3}{4}(8.52) = 6.39 \text{ hr}$$

The perigee altitude of the phased ellipse is

$$h_{PF} = 2\,a_F - h_{AF} - 6880 = 18{,}840 - 8000 - 6880 = 3960 \text{ naut miles}$$

where

$$a_F = a_T(P_F/P_T)^{2/3} = 11{,}440(\tfrac{3}{4})^{2/3} = 9420 \text{ naut miles}$$

and the apogee altitude of the phased ellipse h_{AF} equals 8000 naut miles.

Phased Ellipse Method, Parking Perigee (PE, PP)

This method is similar to the first, except that the phased ellipse is tangent to the parking orbit at perigee rather than tangent to the target orbit at apogee (Fig. 2). At $t = 0$, a velocity increment ΔV_1, sufficient to achieve a preplanned phased ellipse, is added linearly to all of the satellites, and an additional ΔV_2, which will yield a transfer orbit whose apogee is tangent to the target orbit, is added to one of them. At apogee, ΔV_3, which will produce circular orbit velocity, is added to that satellite. At subsequent perigees of the phased ellipse, the remaining satellites are injected, ΔV_2, one by one into the transfer orbit. After $(n - 1)$ revolutions in the phased ellipse plus an interval of time equal to $P_{TR}/2$, the system is established. Thus, Eq. (1) also is valid for this method. Equations analogous to (2) and (3) may be derived easily for this method. Thus

$$a_T \geq 2\,a_F - a_P \quad \text{or} \quad (a_F/a_T) \leq \tfrac{1}{2}[1 + (a_P/a_T)] \qquad (5)$$

$$(P_F/P_T) \leq \{\tfrac{1}{2}[1 + (a_P/a_T)]\}^{3/2} \qquad (6)$$

Given values of a_P and a_T, upper bounds on a_F and P_F may be obtained. Equation (4) still is valid for this method.

Given a_P, a_T, and n, one again determines, from Eq. (4), the allowable values of P_F/P_T smaller than the upper bound specified by Eq. (6); the smallest of these will yield the minimum time to establish the satellite system. The ΔV_{total} requirement for each allowable P_F/P_T is the same, and it is also equal to that for the first method, PE, TA. For the example previously discussed, the allowable values of P_F/P_T are again $\tfrac{1}{4}$ and $\tfrac{3}{4}$, but now $\tfrac{3}{4}$ is eliminated because the upper bound from Eq. (6) is 0.532. The velocity requirements and time required to establish the system for $P_F/P_T = \tfrac{1}{4}$ are

$\Delta V_1 = 2530$ fps — can be supplied by boost vehicle

$\Delta V_2 = 3400$ fps, $\Delta V_3 = 4390$ fps — must be supplied by satellite

$$T = (4.54/2) + 3\,(2.13) = 8.66 \text{ hr}$$

where

$$P_F = \tfrac{1}{4}P_T = \tfrac{1}{4}(8.52) = 2.13 \text{ hr}$$

The apogee altitude of the phased ellipse

$$h_{AF} = 2a_F - h_{P_F} - 6880 = 9080 - 150 - 6880 = 2050 \text{ naut miles}$$

where

$$a_F = a_T(P_F/P_T)^{2/3} = 11{,}440(\tfrac{1}{4})^{2/3} = 4540 \text{ naut miles}$$

and

$$h_{PF} = 150 \text{ naut miles}$$

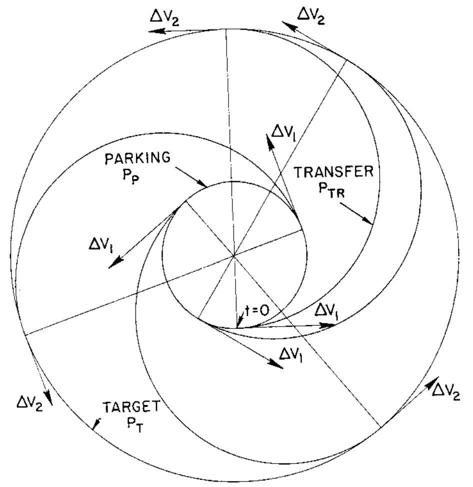

Fig. 3 Schematic of the Hohmann pinwheel method.

Hohmann Pinwheel Method (HP)

This method is best described by referring to Fig. 3. At $t = 0$, one satellite initiates a Hohmann transfer from the parking orbit to the target orbit. After a suitably chosen interval of time, a second satellite initiates a similar transfer. After an interval of time equal to the first, the third satellite initiates its Hohmann transfer, and so on until the last satellite has completed its transfer. Phasing is achieved by means of waiting periods in the parking orbit between satellite expulsions, where

$$t_W = P_P\,P_T/n(P_T - P_P) \qquad (7)$$

The total time required to establish the satellite system is

$$T = (P_{TR}/2) + (n - 1)t_W \qquad (8)$$

The angular travel ϕ in the parking orbit until the last satellite initiates its Hohmann transfer is given by

$$\phi = \frac{360°(n-1)t_W}{P_P} = \frac{360°(n-1)P_T}{n(P_T - P_P)} \qquad (9)$$

For the example previously discussed,

$$t_W = 1.50(8.52)/[4(8.52 - 1.50)] = 0.455 \text{ hr}$$

$$T = (4.54/2) + 3(0.455) = 3.64 \text{ hr}$$

$$\phi = 360\,(3)\,(8.52)/[4(8.52 - 1.50)] = 327.7°$$

Each satellite must supply the entire velocity requirement, 10,320 fps, of the Hohmann transfer.

Fast Pinwheel Method (FP)

This method is identical to the Hohmann Pinwheel method except that a faster transfer orbit is utilized. The initial ΔV is in the direction of vehicle motion but is larger than the initial Hohmann ΔV. The second ΔV, applied at the first intersection of the transfer and target orbits, must transform, in magnitude and direction, the transfer orbit velocity into the target orbit velocity. Thus, a faster transfer is achieved at the expense of a greater ΔV requirement. The tradeoff is shown in Table 1. Note that the waiting

Table 1 Tradeoff between transfer time and ΔV requirements for the FP method

ΔV_{total}, fps	T, hr	ΔV_{total}, fps	T, hr
10,320	2.27 (Hohmann)	12,000	1.67
10,500	2.15	13,000	1.54
11,000	1.91	14,000	1.45
11,500	1.77	15,000	1.39

Table 2 Comparison of methods with $n = 4$ satellites, $h_P = 150$ naut miles, and $h_T = 8000$ naut miles

	ΔV requirements, fps			
Method	Can be supplied by booster	Must be supplied by satellite	Total	Time to establish system, hr
PE, TA	8710	1,610	10,320	21.44
PE, PP	2530	7,790	10,320	8.66
HP	0	10,320	10,320	3.64
FP[a]	0	11,000	11,000	3.28

[a] The values given correspond to an arbitrarily chosen point from Table 1.

period t_W between satellite expulsions is not a function of the transfer orbit method. Therefore, Eq. (7) is valid for the present method as well as the HP method. For the example carried throughout the analysis, a comparison between the four methods just described is presented in Table 2.

Note the variance in time required to establish the system for approximately equal values of ΔV_{total}. The tradeoff is between time required to establish the system and the fraction of the velocity requirements that can be supplied by the boost vehicle.

Reference

[1] Lüders, R. D., "Satellite networks for continuous zonal coverage," ARS J. **31**, 179–184 (1961).

Large Thermal Deflections of Thin-Walled Metal Tubes in Space

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Nomenclature

E = Young's modulus
h = cylinder wall thickness
I = moment of inertia
k = thermal conductivity
l = length of tube
M_0 = moment independent of γ
r = radius of tube
R = radius of curvature of tube
s = arc length
S = solar constant
T = absolute temperature
$\bar{T}$ = mean radiant temperature, $(S\bar{\alpha}/\pi\sigma\bar{\epsilon})^{1/4}$
T_0 = minimum temperature of cross section
α_1 = circumferential conductance parameter, $kh/(r^2\sigma\bar{\epsilon}\bar{T}^3)$
α_2 = longitudinal conductance parameter, $kh/(R^2\sigma\bar{\epsilon}\bar{T}^3)$
$\bar{\alpha}$ = over-all solar absorptivity
β = radiation parameter $\epsilon_i/\bar{\epsilon}$,
γ = angle between local normal and sun vector
ϵ = over-all infrared emissivity
θ = cylindrical coordinate, $0 < \theta < \pi$
ν = coefficient of thermal expansion
σ = Stefan-Boltzmann constant
τ = dimensionless temperature
τ_0 = lowest value of τ at a given cross section

Introduction

Long, thin-walled metal tubes are used on satellites for certain functions, e.g., as booms for gravity-gradient stabilization. Uneven solar heating can cause large thermal deflections in these tubes.

In this note, a procedure is outlined for calculating the deflection for a symmetrical but otherwise arbitrary temperature distribution through each particular cross section. The assumptions are as follows: temperature gradient through the thickness of the tube wall is negligible, internal radiation is of lesser importance than circumferential conduction, for each cross section there is a mean radiant temperature proportional to $\cos^{1/4}\gamma$, and the effects of longitudinal conduction are negligible in comparison with circumferential conduction. The first two assumptions can be justified readily for beryllium tubes of 1-in. diam and 0.005-in. wall thickness by a direct comparison of the parameters α_1 and β. The third assumption is introduced because, when the effects of circumferential conduction predominate, the local temperature should be determined from the relation implied in the Stefan-Boltzmann law. The final assumption is discussed in this paper.

Statement of Problem

The temperature equation can be stated in the dimensionless form as

$$-\alpha_1\frac{\partial^2\tau}{\partial\theta^2} - \alpha_2\frac{\partial^2\tau}{\partial\gamma^2} = \cos\gamma f(\theta) - (1+\beta)\tau^4 + \frac{\beta}{4}(g\gamma)\int_\theta^{\theta+2\pi}\tau^4(\theta')\sin\frac{\theta'-\theta}{2}\,d\theta'$$

where the derivatives indicate the effects of conduction in the circumferential and longitudinal directions, and the terms on the right-hand side are the direct solar-heat input, heat loss by its own radiation, and heat gain by internal radiation. If $0(\partial^2\tau/\partial\theta^2) = 0(\partial^2\tau/\partial\gamma^2)$, and $r \ll R$, then the second term on the left-hand side can be neglected in comparison with the first. Moreover, if $\alpha_1 > \beta$, then $g(\gamma) \approx 1$ can be set, and if $\alpha_1 \gg 1$, the problem reduces to the solution of the linear integrodifferential equation for a tube inclined at an angle of $90° + \gamma$ with respect to sun rays.[1] This solution depends upon the Fourier series representation of the solar-heat input function $S\cos^+\theta\cos\gamma$ with $\cos^+\theta$ = half-wave rectified cosine function:

$$\cos^+\theta = \frac{1}{\pi} + \frac{1}{2}\cos\theta + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n\cos 2n\theta}{1-4n^2}$$

To show the effect of a variable γ, one can here "delinearize" the solution from Ref. 1 by writing $4\tau - 3 \approx \tau^4$; then, after a few transformations, one has finally $\Delta\tau(\theta,\gamma) = \Delta\tau(\theta)\cos^{1/4}\gamma$. The solution may be rewritten as

$$(\tau-\tau_0)(\cos\gamma)^{-1/4} = \left(\frac{\pi}{8}a_0 + \frac{\pi}{4}\sum_{n=1}^{\infty}a_n\cos n\theta\left\{\frac{1}{4}\alpha_1 n^2 + [1-(1+\beta)4n^2](1-4n^2)^{-1}\right\}^{-1}\right)\int_\pi^\theta \quad (1)$$

As long as $R \gg r \gg h$ holds, the simple theory of strength of materials may be expected to give correct results, so that the equation of the elastic line (Fig. 1) will be $d\gamma/ds = M/EI$. Here the moment M is caused entirely by the thermal loading. In view of Eq. (1), let $M_0 = M/(EI\cos^{1/4}\gamma)$, or

$$M_0 = \frac{\nu}{I}\int_A (T-T_0)y\,dA \quad (2)$$

Equation (2) neglects the direct effect of solar pressure†; since

$$d\gamma/ds = M_0\cos^{1/4}\gamma \qquad \text{for } \gamma_0 = 0 \qquad \text{and } s(0) = 0$$

$$\int_0^\gamma \sec^{1/4}\eta\,d\eta = M_0 s \quad (3)$$

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† For small deflections, the effect of solar pressure can be calculated by the expression $y = prl^4/4EI$.